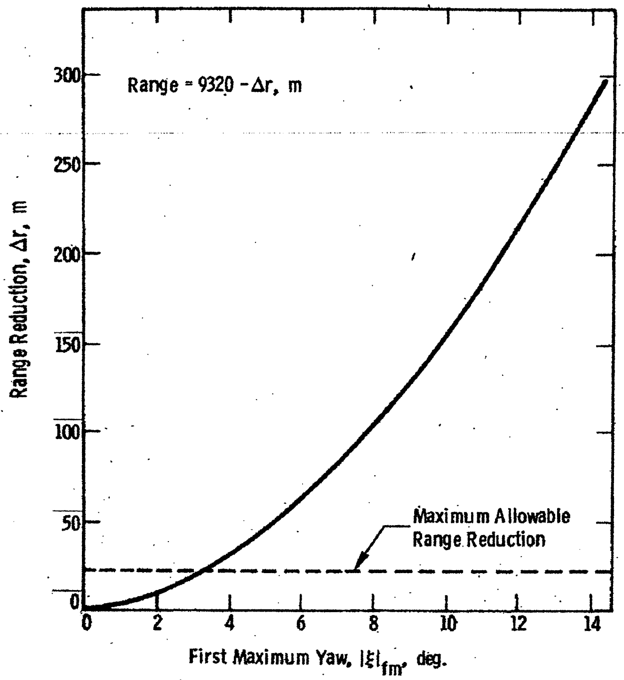

Fig. 3 Effect of first maximum yaw on range reduction.

reduce sharply to 0.5. Note in Fig. 1b that for this case the maximum magnification value occurs for $S_g = 1.0009$.

6-DOF Motion Simulations

To demonstrate the effects that K_3 has on the magnitude of the initial maximum total angle of attack (first maximum yaw), six-degree-of-freedom (6-DOF) flight simulations were obtained for an inertially asymmetric (principal axis misalignment and *cg* offset effects included) 8-in. artillery projectile. For a transonic low gyroscopic stability firing condition, the magnitude of K_3 was varied over the range 0 to 0.3 deg. With the exception of the asymmetries that produce K_3, all other physical characteristics of the projectile remained unchanged for the simulations. A perfect exit from the gun muzzle was assumed $\dot{\xi}_0 = \xi_0 = 0$; therefore, the initial disturbances result from nonzero $\ddot{\xi}_0$ caused by K_3.

The 6-DOF results presented in Fig. 2 confirm the large mangification of the trim angle in the initial transient angular motion. As shown, the analytical results provide an upper bound on the trim induced first maximum yaw ($|\xi|_{fm}$, Fig. 2). In addition to showing that small trim angles can produce very large initial disturbances, these results also indicate that small differences in the level of mass asymmetry between otherwise identical projectiles can have a large effect on the relative magnitudes of their initial angular motion.

For transonic firing conditions, damping is often extremely sluggish. Because of angle-of-attack induced drag and sluggish damping, the magnitude of the initial total transient angle of attack can have a relatively large effect on range through its effect on ballistic coefficient. This was found to have a much greater impact on the accuracy requirements than the trajectory deflections caused by the initial disturbances. For the selected transonic firing condition, the results presented in Fig. 3 demonstrate the large effect of induced drag on range. Note in Fig. 3 that reductions in range of 24, 60, and 138 m result for first maximum yaw values of 3.1, 6.0, and 8.9 deg. respectively. These first maximum yaw values were produced by K_3 values of 0.1, 0.2, and 0.3 deg (Fig. 2). All of these reductions in range exceed a 0.25% of range accuracy requirement, a representative requirement for ballistic similitude. These results indicate that small differences in the mass asymmetries of projectiles that are otherwise identical could result in failure to satisfy specified ballistic similitude requirements. Therefore the effects of small mass asymmetries must be considered when attempting to ballistically match projectiles.

Conclusions

Small mass asymmetries (principal axis misalignment and lateral *cg* offset) have proportionately large effects on the magnitude of artillery projectile first maximum yaw. The first maximum yaw levels produced by these asymmetries can cause unacceptable changes in range. After satisfying even the fundamental ballistic similitude criterion (identical shape, weight, static margin, and moments of inertia), it may still be necessary to reduce small differences in mass asymmetries between projectiles to insure ballistic similitude.

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Launcher Length for Sounding-Rocket Point-Mass Trajectory Simulations

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Introduction

THE simulation of sounding-rocket trajectories is basic to all other simulative exercises required for mission planning and hardware design. Because it is the cornerstone of all analytical efforts, large numbers of computer runs are required for this purpose.

Most of these simulations are made with a point-mass trajectory model in which the rigid-body rotational degrees of freedom are solved a priori, typically by assuming that the rocket always heads instantly into the relative wind. Such a model is extremely useful because it requires minimal effort in the preparation of input data, and because its running times and costs are low.

In reality, rockets have dynamical behavior corresponding to many modes associated with a large number of degrees of freedom. The point-mass trajectory model is a filtering approximation which tends to replicate the long period (Earth orbital period, if the rocket could pass through the solid Earth) or phugoid motion fairly well, while suppressing the dynamics of the short period motion, aeroelastic motions, etc., all of which have characteristic periods much shorter than the phugoid.

The point-mass model breaks down near the launch when the phugoid-short period intermodal coupling is especially large. The point-mass equations have a singularity there which is not present in the full set of six rigid-body equations.

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The correct way to look at this is to consider it as a singular perturbation problem, whose outer solution is given by the point-mass results.

It has long been known that if an artificial launcher length, longer than the real physical launcher length is used the singularity in the point-mass equations can be avoided and reasonable looking results obtained. The purpose of this Note is to solve the singular perturbation problem and to develop an artificial launcher length, for use in the point-mass model, which will provide valid solutions everywhere except very near the launcher.

Solution

The point of departure is the solution given[1] for the gravity turn. Since gravity acts as an Earth fixed disturbance, the solution to the inner problem can be found on a planar, nonrolling basis. The simplest dynamical model is based on the body axis equations

$$\dot{w} - \dot{\theta}u = g\sin\gamma \tag{1}$$

$$I\ddot{\theta} = M_\alpha \alpha \tag{2}$$

$$M_\alpha = -I\lambda u^2 \tag{3}$$

$$\alpha \cong \frac{w}{u} \tag{4}$$

$$\gamma = \theta - \alpha \tag{5}$$

where g = acceleration due to gravity; $u, w = x$ and z velocity components; γ = flight path angle, I = pitch moment of inertia; λ = initial pitch wave number, and θ = inertial pitch angle. Assuming constant axial acceleration, and replacing $\sin\gamma$ by its initial value γ_0 in Eq. (1), the resulting flight path angle history near launch is given[1] as

$$\frac{\gamma_3}{\gamma_0}(s) = 1 + \frac{g}{a}\left[\frac{\pi}{2\lambda s}\right]^{1/2}\Big[\sin\lambda s\,(S(\lambda s) - S(\lambda L)) + \cos\lambda s\,(C(\lambda s) - C(\lambda L))\Big] + \pi\frac{g}{a}\Big[J(\lambda s) - J(\lambda L) + C(\lambda s)S(\lambda L) - C(\lambda L)S(\lambda s)\Big] \tag{6}$$

where L = launcher length, s = distance along flight path,

$$C(x) + iS(x) = \int_0^x \frac{e^{it}}{[2\pi t]^{1/2}}\,dt = \text{Fresnel integrals, and}$$

$$J(x) = \int_0^x \frac{\sin t\,C(t) - \cos t\,S(t)}{[2\pi t]^{1/2}}\,dt$$

Note that γ_3 is the flight path angle solution obtained from the above three degree of freedom (DOF) analysis. The corresponding two degrees of freedom, or zero α point mass, solution is given by

$$\frac{\gamma_2}{\gamma_0}(s) = 1 + \frac{g}{2a}\log\frac{\lambda s}{\lambda L} \tag{7}$$

for small s.

The outer limit of the inner solution is found using results obtained from Ref. 2. When the inner limit of the outer solution, Eq. (7), is matched to the outer limit of the inner solution, both show the logarithmic form previously indicated, demonstrating procedural validity.

The matching of the two solutions leads to the result for the launcher length L_2 to be used in 2 DOF, or point mass, simulations which will have the same asymptotic behavior as 3 DOF solutions for launcher length L_3. It is found to be

$$\frac{L_2}{L_3} = \frac{1}{4\lambda L_3}\exp\Big\{-\Gamma + 2\pi J(\lambda L_3) + \pi[C(\lambda L_3) - S(\lambda L_3)]\Big\} \tag{8}$$

where $\Gamma = 0.5882157...$ which is Euler's or Mascheroni's constant.

Table 1 Incremental launcher length

λL_3	$\lambda(L_2 - L_3)$
0.000	0.1403
0.001	0.1509
0.002	0.1550
0.005	0.1624
0.010	0.1702
0.020	0.1796
0.050	0.1939
0.100	0.2038
0.200	0.2085
0.500	0.1967
1.000	0.1678
2.000	0.1245

Results

Equation (8) may be evaluated numerically using the integral tables of Ref. 1. When this is done, it is found that the dimensionless increment in launcher length, $\lambda(L_2 - L_3)$, is the most nearly constant of the output variables. The numerical data are displayed in Table 1. These calculations place the customary practice of using a larger than life size launcher in point-mass trajectory simulations on a firm theoretical foundation.

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Imperfection Sensitivity and Isoperimetric Variational Problems in Stability Theory

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Introduction

THE investigation of shell buckling has occupied an increasingly prominent place in aeronautical and

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